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Do Calculators Support Inductive Thinking?

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Abstract
In many countries, calculators that are considered traditional technological devices are widely used in compulsory mathematics education, but the creative application is almost missing from everyday Hungarian school practice. In this paper we focus on the inductive observation process supported by calculator and investigate whether the way of observation have an influence on the way of reasoning. We designed a problem appropriate for the „looking for pattern" strategy and constructed a lesson plan for five experimental groups: four of prospective mathematics teachers and one of 6th Grader students. We summarise our findings on pros and cons of using calculator in the inductive thinking process.

1. Introduction

Nowadays the term "technology" in the context of school mathematics mainly refers to computers, tablets or smartphones. In many countries, calculators that are considered traditional technological devices are widely used in compulsory mathematics education (in the classroom as well as in exam situation). At the same time, it is observed that the way of use is developing. While earlier it was used to support basic calculations only, now it has an increasing role in the construction of knowledge too.

In Hungary, the calculator rarely appears in the mathematics classroom in lower secondary schools (Grades 5-8). Here, the emphasis is on mental or paper and pencil calculations. Students in upper secondary schools (Grades 9-12) use the calculator in the classroom as well as on the final exam, but their use is strictly limited, graphing and CAS calculators are not allowed. Students perform simple calculations concerning one or two operations or determine the values of trigonometric or logarithmic functions (instead of the former printed tables). We argue that creative application of calculators is almost missing from everyday Hungarian school practice.

Our recent research is embedded into the project *Content Pedagogy Research Program of the Hungarian Academy of Sciences* which, among others, aims at the revitalization of experiments for teaching complex mathematics conducted by Tamás Varga in 1960—1980. The main points of the former program were: (1) problem-based learning; (2) guided discovery or learning by discovery; (3) provoking classroom discussion; (4) manipulatives, different tools (e.g. calculator) [12]. After 50 years, when calculators are more widely used not only in schools but in everyday life, it is worth redefining its role as educational tool in mathematics education. In this paper we investigate the role of calculators in the inductive reasoning process based on a problem from the field of number theory.

2. Theoretical background

The term problem-based learning (PBL) is used as one of the active learning strategies [9], which means that students learn about a topic through the solving of problems. "PBL is an instructional (and curricular) learner-centred approach that can empower learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem." [10, p. 12]. As Pólya wrote in his book the principle of active learning is the following: "For efficient learning, the learner should discover by himself as large a fraction of the material to be learned as feasible under the given circumstances." [9, p. 103]. Tamás Varga's former teaching experiment relied on the discovery-based learning style, which is strongly related to the PBL approach [1].

The implementation of the above method in mathematics classrooms requires - among others - inductive activities. Haverty, Koedinger, Klahr and Alibali argue that fundamental areas of inductive activities are data gathering, pattern finding and hypothesis generation [3]. More precisely Yeo and Yeap distinguish two parts of induction: (1) inductive observation and (2) inductive reasoning [13]. The observation part consists three phases such as observation of particular cases; formulating a general conjecture; testing it by other particular cases. The inductive reasoning phase means that students use the underlying mathematical structure to argue that the observed rule will always work. Mason, Burton and Stacey formulate this expression more accurately: the inductive reasoning should be a rigorous proof or justification [7].

There is no doubt that digital technologies today are present in students' everyday life and in schools as well. However, the technology the teacher chooses, depends on his/her educational aims, learning methods he/she wants to use, the level of digital literacy of students as well as of teacher himself, and last but not least on the availability of these technologies [6]. Despite the fact that calculators are available to all students, their use in mathematics classrooms is quite controversial among primary and secondary school teachers. The survey conducted in Slovakia by Korenova showed that 56% of the elementary and middle school teachers thought that calculators should be used during mathematics classes [6].

Kissane highlights the usability of calculators in problem solving processes [5]. "Experience in many countries suggests that calculators are important for more than calculation, however, and also that mathematics consists of a great deal more than calculation." [5, p. 2]. Thompson and Sproule distinguish product- and process-oriented pedagogical goals and argue that calculators have an important role both in product- and process-oriented activities [11]. In contrast to product-oriented activities, where the goal is the computation itself, in process-oriented activities the computations help students discover patterns and test conjectures and so allow them to concentrate on the processes itself. Hembree and Dessart's meta-analysis also showed that using a calculator in problem solving more often resulted in selection of a proper approach to a solution [4]. The process, when the calculator becomes a mathematical work tool (instrumental genesis) was analysed by Guin and Trouchet [2]. They pointed out that this "depends on the tool's constraints and potentialities, on students' knowledge, and on the class' work situations." [2, p. 204] Therefore it is important to understand how (simple, scientific or graphing) calculators contribute to the learning of mathematics, how they can be used effectively for enhance problem solving both in elementary and in secondary school. In this study we focus on the inductive observation process supporting by the calculator.

Guin and Trouche also describe how students use technology by introducing five types of usage:
- *theoretical type* is based on paper and pencil work and systematic use of mathematical references where calculator is used to verify final result only;
- *rational work* method, characterized by a reduced use of calculator, mainly working within traditional environment;
- *random type* method, characterized by trial and error procedures with very limited references to understanding tools and without verifying strategies of machine results;

- *mechanical work* method, characterized by information sources restricted to the calculator investigations and simple manipulations. Reasoning is based on the accumulation of consistent machine results;
- *resourceful work* method, characterized by an exploration of all available information sources (calculator, but also paper/pencil work and some theoretical references) In resourceful work method calculations are paper/pencil performed and performed with the calculator at the same time [2].

3. Research question

Proof and argumentation are included in curricula for 5-12 graders in Hungary; however, we find little emphasis on them in classrooms. Despite the statements of curriculum it seems that the teachers' dilemma still remains; what to teach for students: following routine procedures vs. reasoning and proving. In addition only traditional tools of mathematics learning are frequently used in the classrooms: paper, pencil and calculator for one or two steps calculations. All of these facts encouraged us to formulate the following research question:

What is the impact of using calculator in the inductive thinking process?

4. Methodology

We designed a problem appropriate for the "looking for pattern" strategy using the calculator. In this paper we describe its different classroom implementations. A lesson plan was constructed for two experimental groups: (1) prospective mathematics teachers and (2) 6^{th} Grader students in order to try our ideas.

The original problem is the following: the product of two numbers ending in 76 ends in 76. This problem is surely part of the mathematical tradition. We learnt it from the Hungarian translation of the classical Russian book by Y. I. Perelman appeared in 1955 in Hungary [8]. We restricted the statement to square numbers and formulated a problem sequence:

1. What are the *one-digit numbers*, whose square ends in the same digit? (Answer: 0; 1; 5; 6.)
2. What are the *two-digit numbers*, whose square ends in the same two-digits? (Answer: 25; 76.)
3. What are the *numbers* whose last two digits are the same as the last two digits of their square? Explain your solution. (Answer: ∗00; ∗01; ∗25; ∗76.)

Table 1 shows the participants and the circumstances of our experimental study. We analyzed the video and sound recordings by qualitative interpretive analysis method, based on recognizing common patterns in students' activities in different classrooms.

Table 1: The sample and the circumstances

	Group A	**Group B**	**Group C**	**Group D**	**Group E**
Participants	Prosp. teachers (Grade III.)	Prosp. teachers (Grade IV.)	Prosp. teachers (Grade IV.)	Prosp. teachers (Grade III.)	6^{th} Grader students
Number of students	5	7	10	16	14
Place of the lesson	University of Nyíregyháza	University of Nyíregyháza	University of Nyíregyháza	University of Debrecen	Elementary school in Nyíregyháza

Teacher	the second author	the second author	the second author	the first author	the second author
Date of the lesson	21/03/2018	24/03/2018	19/04/2018	10/04/2018	14/05/2018
Documents	audio recording, written notes	audio recording, written notes	audio recording, written notes	audio recording, written notes	video recording, written notes

5. Findings and discussion

The students worked first individually then in pairs. After some minutes we summarized their answers in a form of classroom discussion.

In this paper we focus only on Question 3. The calculator was available for the students and they used it in the inductive observation phase. In particular, we were interested in students' reasoning process, whether their explanation is algebraic i.e. $(100A + 76)^2 = 100B + 76$ or arithmetical i.e. they use the long multiplication algorithm of $*76 \times *76$.

We recall some episodes from the classroom implementations. These episodes focus only on the numbers ending with 76.

5.1. Group A

The prospective teachers looked for the pattern using the strategy of sample trial as it is seen on Figure 1.

Figure 1: Sample trial

Teacher: $76 \cdot 76 =$? Try another number ending with 76. [Students shared their examples.]
Teacher: What is your conjecture?
Students: The square of a number ending with 76 ends in 76 too.
Teacher: Why? Try to explain. Is this statement *true in general*?

In the justification part of this teaching episode there was a student, *Levente* who was not interested in the explanation; he wanted to find other numbers with the same property, so he checked more numbers. Another student, *Veronika* referred to the long multiplication algorithm: "Actually it doesn't matter how many digits the number has, the last two digits of the square depend only on the last two digits of the number itself." Her arithmetical reasoning was approved by the teacher, who after that suggested formulating general algebraic expression for the numbers ending with 76: "How can we *write all the numbers ending with 76 in one expression*?" The students couldn't understand the question and weren't able to think algebraically concerning this problem.

5.2. Group B and C

Answering Question 2, the prospective teachers used the sample trial strategy first. Some of them looked for appropriate numbers randomly, while others recognised that they should test only the numbers ending with 0, 1, 5 or 6. In answer to question 3, the random testing strategy was used on the teacher's suggestion. The group knew the command *RanInt* from the previous lesson, so they applied it in a proper way.

Teacher: Write digits before 76. Do the required calculation. [Sample trial]
…
Teacher: Try more numbers in order to *check our conjecture*. How is it possible to generate random numbers ending with 76? [suggestion for the *RanInt* command]
…
Teacher: How can we prepare such kind of number e. g. from the 336?
Pali: We write 76 *behind them*.
Teacher: What does Pali do *mathematically*?
Judit: He multiplies 336 by 100 and adds 76 to it.

Before the reasoning phase they generated many numbers by pushing the "=" key (Figure 2) and investigated them. After the teacher's question: "Can we prove what we discovered before?" one of the students, Pali noticed that there is no need to justify, because he "believes in the calculator". *Misi* formulated a new observation "The digit before 76 is always an odd number." instead of justification of the original one. When the teacher reminded the students to the way they generated random numbers, all but one student could construct and interpret the expression: $(100A + 76)^2 = 100^2A^2 + 200A \cdot 76 + 76^2$. At this point the teacher returned to Misi's unexpected observation and asked him to prove it using the formula above.

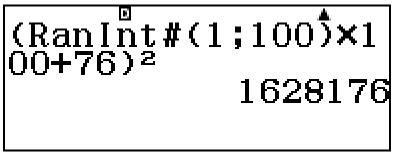


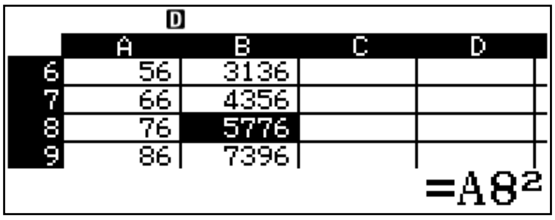
Figure 2: Randomized trial

5.3. Group D

Students of Group D used further advanced operations of the Casio ClassWizz calculator in order to apply the systematic trial strategy:

Teacher: "What are the tens digits of the square numbers ending with 6? Check the squares of two-digit numbers. Use the *Fill formula* option in the *Spreadsheet* mode."

They constructed a table containing the squares of the two-digit numbers ending with 6 (Figure 3). When they worked with the *Fill formula* command, computational way of thinking also appeared (see Figure 4, where A1=6 and A2:A10 are the cells of the first column in the table.).



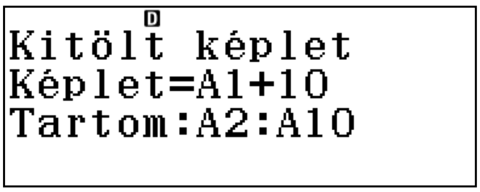


Figure 3: The table of investigated numbers Figure 4: Recursive formula

In the inductive observation stage the teacher came up with a new problem first: "Whether the square of *any number* ending with 76 ends in 76?", then suggested the observation of particular cases: "Check the *3-digits numbers* ending with 76." The students used the *Fill formula* command again and formulated correct conjecture.

The two problems, i.e. checking the squares of three-digit numbers or any numbers ending with 76 help to highlight the similarities and the differences between systematic trial and proof by exhaustion methods. Proof by exhaustion (proof by cases), is a method of mathematical proof in which the statement to be proved is split into a finite number of cases and each type of case is checked ($176^2; 276^2; 376^2; 476^2; 576^2; 676^2; 776^2; 876^2; 976^2$). Some of the students was not sure, whether after trying all the 3-digits numbers ending with 76 is the proof for any numbers necessary or not. However upon teacher's request, they were able to justify it arithmetically as well as algebraically.

5.4. Group E

In the observation phase 6th Grader students tested 4-5 numbers ending with 76 (sample trial, Figure 5). 7 students out of 14 formulated the conjecture after collecting some examples (Figure 6).

Figure 5: Testing the numbers

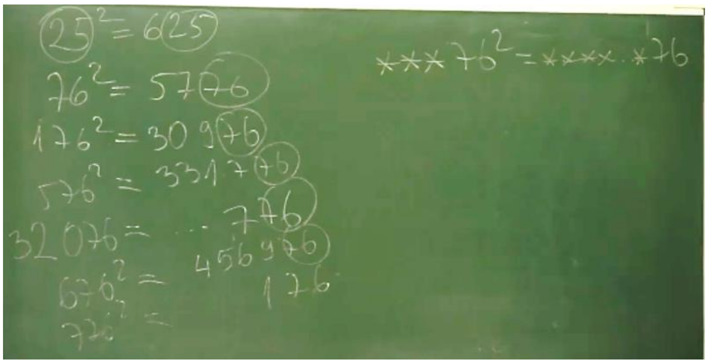
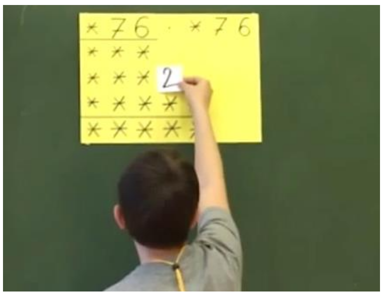


Figure 6: Summarizing the result of testing

In the teacher-guided inductive reasoning phase the students had to figure out some of the missing numbers in order to apply the long multiplication algorithm for the arithmetical explanation.

Bence: $7 \cdot 6 = 42$, so the last digit in this row is 2 (Figure 7).
Eszter: The last 2 digits should be 76 and 2+5=7, so I put here 5 (Figure 8).

The teacher interrupted Eszter, because she wanted to use the statement which was required to prove. After the teacher's explanation Eszter understood the mistake and answered the question: "Which digit can you calculate easily?" (Figure 9), finally she justified the statement (Figure 10).



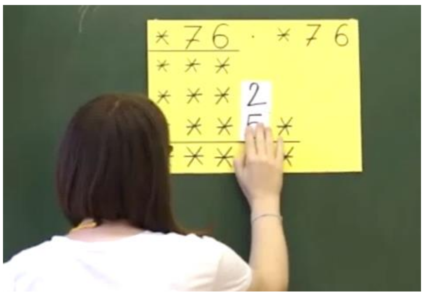


Figure 7: Bence: “The last digit is 2.”

Figure 8: Eszter: “I put here 5.”

Figure 9: Eszter: “6·6=36”

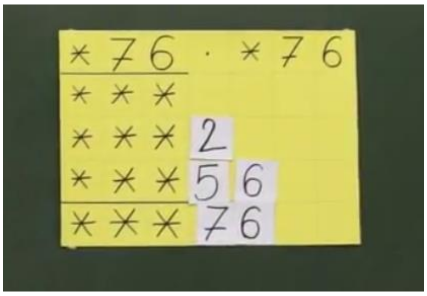


Figure 10: Justification by long multiplication

6. Conclusion

We summarise our findings according to our research question on impact of using calculator in the inductive thinking process.

Calculators support discovery based learning by quickly observing many particular cases. Therefore formulating conjecture was quite easy for every student regardless of age. It could also mean that every student is able (at least) to start the problem solving process by using the calculator. The way of looking for particular cases influences the reasoning process: randomized trial using the *RanInt* command seems to support the algebraic proof of the statement. Systematic trial using the *Spreadsheet* mode of the calculator helps to develop computational as well as algebraic thinking. These findings are consistent with the claims of Hembree and Dessart [4] and Guin and Trouche [2] explained in the Theoretical background section.

At the same time, the memory limit of the calculator may obstruct the investigation. Another observation that can be categorized as a disadvantage of the calculator is that sometimes it seems easier to press the buttons quickly instead of thinking which numbers are worth checking out.

Additionally, we have observed some phenomena that we believe are not directly related to the use of the calculator, but they help us to better understand students’ inductive reasoning process: (1) The difference between the conjecture and the proved statement is not necessarily clear for the 6th Graders. (2) It is hard to understand, even for prospective teachers, that after checking the conjecture with further particular cases we only support our conjecture but don’t prove it mathematically. (3) It is not obvious for the students that proof by cases differs from trial of some (but not every) cases.

We should also remark, that students sometimes come back with new findings in the inductive observation phase so the teacher has to react in real time. It requires advanced professional knowledge in one hand and to change his/her previous teaching plan immediately on the other hand.

Our research revealed the following types of students’ behaviour according to [2].

- Theoretical type: Eszter (Group E) did not use calculator, only paper and pencil.
- Rational type: Calculator is used for experience, but reasoning by long multiplication algorithm is basically independent of machine results. (Eg. Veronika in group A.)
- Random type: In the second question some students did not use the result of the first question, but randomly tried two digits numbers.
- Mechanical type: The reasoning is the accumulation of results from the calculator. Levente (Group A) and Pali (Group B) behaved like this because they did not want to understand the results of the calculator.

- Resourceful type: The answer is conjectured from the results computed with the calculator that also influenced the paper and pencil reasoning work. This was the most typical behaviour in our experience.

In summary we argue, that the use of the calculator can support inductive thinking processes in the phase of observation of particular cases and testing the conjecture. We also found that the way students use the calculator in the observation phase influences the way of their reasoning. It also became clear that the use of calculator requires new attitudes from students and teachers.

7. Acknowledgements

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